Halos in a deformed Relativistic Hartree-Bogoliubov theory in continuum

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Abstract. In this contribution we present some recent results about neutron halos in deformed nuclei. A deformed relativistic Hartree-Bogoliubov theory in continuum has been developed and the halo phenomenon in deformed weakly bound nuclei is investigated. These weakly bound quantum systems present interesting examples for the study of the interdependence between the deformation of the core and the particles in the halo. Magnesium and neon isotopes are studied and detailed results are presented for the deformed neutron-rich and weakly bound nuclei ^{42}Mg. The core of this nucleus is prolate, but the halo has a slightly oblate shape. This indicates a decoupling of the halo orbitals from the deformation of the core. The generic conditions for the existence of halos in deformed nuclei and for the occurrence of this decoupling effect are discussed.

INTRODUCTION

Halo is one of the most interesting exotic nuclear phenomena. In halo nuclei, the extremely weakly binding property results in many new features, e.g., the coupling between bound states and the continuum due to pairing correlations and very extended spatial density distributions. Therefore one must consider properly the asymptotic behavior of nuclear densities at large distance r from the center and treat in a self consistent way the discrete bound states, the continuum and the coupling between them in order to give a proper theoretical description of the halo phenomenon. This can be achieved by solving the non-relativistic Hartree-Fock-Bogoliubov (HFB) [1, 2, 3] or the relativistic Hartree Bogoliubov (RHB) [4, 5, 6, 7, 8] equations in coordinate (r) space which can fully take into account the mean-field effects of the coupling to the continuum.

For spherical nuclei, relativistic Hartree-Bogoliubov theories in coordinate space have been developed [4, 5, 6, 7, 8]. With the relativistic continuum Hartree-Bogoliubov (RCHB) theory [7, 8], properties of the halo nucleus ^{11}Li has been reproduced quite well [4] and the prediction of giant halos in light and medium-heavy nuclei was made [9, 10, 11]. The RCHB theory has been generalized to treat the odd particle system [12] and

combined with the Glauber model, the charge-changing cross sections for C, N, O and F isotopes on a carbon target have been reproduced well [13].

Since most open shell nuclei are deformed, the interplay between deformation and weak binding raises interesting questions, such as whether or not there exist halos in deformed nuclei and, if yes, what are their new features. In order to consider properly the asymptotic behavior of nuclear densities at large r and to make the numerical procedure less complicated, the Woods-Saxon basis has been proposed in Ref. [14] as a reconciler between the harmonic oscillator basis and the integration in coordinate space. Over the past years, lots of efforts have been made to develop a deformed relativistic Hartree theory [15] and a deformed relativistic Hartree Bogoliubov theory in continuum (the DefRHBC theory) [16, 17, 18, 19, 20]. In order to describe the exotic nuclear structure in unstable odd-A or odd-odd nuclei, the DefRHBC theory has been extended to incorporate the blocking effect due to the odd nucleon(s) [21]. The deformed relativistic Hartree-Bogoliubov theory in continuum with the density-dependent meson-nucleon couplings is developed recently [22].

The halo phenomenon in deformed nuclei has been investigated with the DefRHBC theory [19, 20]. In some deformed neutron-rich and weakly bound nuclei, e.g., ^{42,44}Mg, a decoupling of the halo orbitals from the deformation of the core has been predicted. In this contribution, the results of the DefRHBC theory on the study of deformed halo nuclei will be presented.

THEORETICAL FRAMEWORK

The Dirac Hartree Bogoliubov (RHB) equation for the nucleons reads [23],

$$\int d^3 \boldsymbol{r}' \begin{pmatrix} h_D - \lambda & \Delta \\ -\Delta^* & -h_D + \lambda \end{pmatrix} \begin{pmatrix} U_k \\ V_k \end{pmatrix} = E_k \begin{pmatrix} U_k \\ V_k \end{pmatrix}, \tag{1}$$

where E_k is the quasiparticle energy, λ is the chemical potential, and h_D is the Dirac Hamiltonian,

$$h_D(\boldsymbol{r}, \boldsymbol{r}') = \boldsymbol{\alpha} \cdot \boldsymbol{p} + V(\boldsymbol{r}) + \beta(M + S(\boldsymbol{r})). \tag{2}$$

with scalar and vector potentials

$$S(\boldsymbol{r}) = g_\sigma \sigma(\boldsymbol{r}), \tag{3}$$

$$V(\boldsymbol{r}) = g_\omega \omega^0(\boldsymbol{r}) + g_\rho \tau_3 \rho^0(\boldsymbol{r}) + e \frac{1 - \tau_3}{2} A^0(\boldsymbol{r}). \tag{4}$$

In the particle-particle (pp) channel, we use a density dependent zero range force,

$$V^{\mathrm{pp}}(\boldsymbol{r}_1, \boldsymbol{r}_2) = V_0 \frac{1}{2}(1 - P^\sigma)\delta(\mathbf{r}_1 - \mathbf{r}_2)\left(1 - \frac{\rho(\boldsymbol{r}_1)}{\rho_{\mathrm{sat}}}\right). \tag{5}$$

$\frac{1}{2}(1 - P^\sigma)$ projects onto spin $S = 0$ component in the pairing field. The pairing potential then reads,

$$\Delta(\boldsymbol{r}) = V_0(1 - \rho(\boldsymbol{r})/\rho_{\mathrm{sat}})\kappa(\boldsymbol{r}), \tag{6}$$

and we need only the local part of the pairing tensor

$$\kappa(\boldsymbol{r}) = \sum_{k>0} V_k^\dagger(\boldsymbol{r}) U_k(\boldsymbol{r}). \tag{7}$$

For axially deformed nuclei with the spatial reflection symmetry, we expand the potentials $S(\boldsymbol{r})$ and $V(\boldsymbol{r})$ in Eq. (2) and various densities in terms of the Legendre polynomials [24],

$$f(\boldsymbol{r}) = \sum_\lambda f_\lambda(r) P_\lambda(\cos\theta), \ \lambda = 0, 2, 4, \cdots, \tag{8}$$

with an explicit definition of $f_\lambda(r)$.

The quasiparticle wave functions U_k and V_k in Eq. (1) are expanded in the Woods-Saxon basis [14]:

$$U_k(\boldsymbol{r}sp) = \sum_{n\kappa} u_{k,(n\kappa)}^{(m)} \varphi_{n\kappa m}(\boldsymbol{r}sp), \tag{9}$$

$$V_k(\boldsymbol{r}sp) = \sum_{n\kappa} v_{k,(n\kappa)}^{(m)} \bar{\varphi}_{n\kappa m}(\boldsymbol{r}sp). \tag{10}$$

$\bar{\varphi}_{n\kappa m}(\boldsymbol{r}sp)$ is the time reversal state of $\varphi_{n\kappa m}(\boldsymbol{r}sp)$. Because of the axial symmetry the z-component m of the angular momentum j is a conserved quantum number and the RHB Hamiltonian can be decomposed into blocks characterized by m and parity π. For each m^π-block, solving the RHB equation (1) is equivalent to the diagonalization of the matrix

$$\begin{pmatrix} \mathscr{A} - \lambda & \mathscr{B} \\ \mathscr{B}^\dagger & -\mathscr{A}^* + \lambda \end{pmatrix} \begin{pmatrix} \mathscr{U}_k \\ \mathscr{V}_k \end{pmatrix} = E_k \begin{pmatrix} \mathscr{U}_k \\ \mathscr{V}_k \end{pmatrix}, \tag{11}$$

where

$$\mathscr{U}_k = \left(u_{k,(n\kappa)}^{(m)} \right), \ \mathscr{V}_k = \left(v_{k,(n\kappa)}^{(m)} \right), \tag{12}$$

and

$$\mathscr{A} = \left(h_{D(n\kappa)(n'\kappa')}^{(m)} \right) = \left(\langle n\kappa m | h_D | n'\kappa', m \rangle \right), \tag{13}$$

$$\mathscr{B} = \left(\Delta_{(n\kappa)(n'\kappa)}^{(m)} \right) = \left(\langle n\kappa m | \Delta | \overline{n'\kappa', m} \rangle \right). \tag{14}$$

Further details can be found in the appendixes of Ref. [20].

RESULTS AND DISCUSSIONS

We next present some results from the DefRHBC theory by taking magnesium isotopes as examples and discuss in details results for the deformed neutron-rich and weakly bound nucleus [42]Mg [18, 19, 20].

Magnesium isotopes have been studied extensively in Refs. [18, 20] with the deformed relativistic Hartree-Bogoliubov theory in continuum and the parameter sets
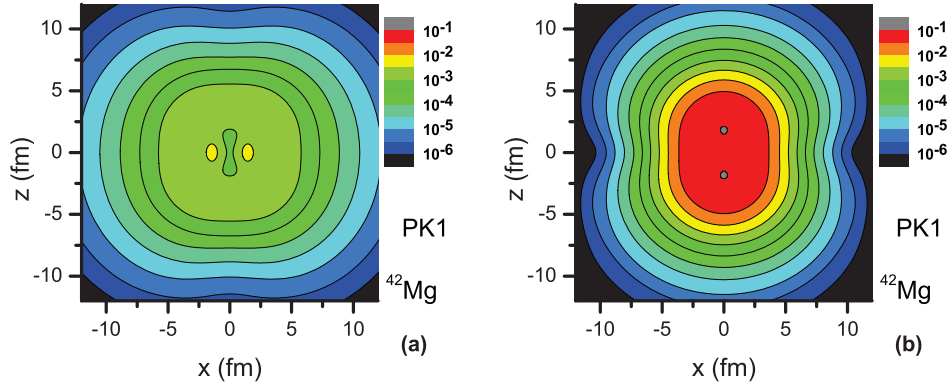
FIGURE 1. (Color online) Density distributions of the ground state of ^{42}Mg with the z axis as the symmetry axis: (a) the neutron halo, and (b) the neutron core. Taken from Ref. [20].

NL3 [25] and PK1 [26]. In the calculations based on the parameter set PK1, ^{42}Mg is the last bound nucleus in Mg isotopes [20].

It was found in Ref. [20] that the ground state of ^{42}Mg is well deformed with a quadrupole deformation $\beta \approx 0.41$, and a very small two neutron separation energy $S_{2n} \approx 0.22$ MeV. In the tail part, the neutron density extends more along the direction perpendicular to the symmetry axis. The density distribution is decomposed into contributions of the oblate "halo" and of the prolate "core" in Fig. 1. The density distribution of this weakly bound nucleus has a very long tail in the direction perpendicular to the symmetry axis which indicates the prolate nucleus ^{42}Mg has an oblate halo and there is a decoupling between the deformations of the core and the halo.

The single particle spectrum around the Fermi level for the ground state of ^{42}Mg is shown in Fig. 2 [20]. The good quantum numbers of each single particle state are also shown. The occupation probabilities v^2 in the canonical basis have BCS-form [27] and are given by the length of the horizontal lines in Fig. 2. The levels close to the threshold are labeled by the number i according to their energies, and their conserved quantum number Ω^π as well as the main spherical components are given at the right hand side. The neutron Fermi level is within the pf shell and most of the single particle levels have negative parities. Since the chemical potential λ_n is close to the continuum, orbitals above the threshold have noticeable occupations due to the pairing correlations. The single neutron levels of ^{42}Mg can be divided into two parts, the deeply bound levels ($\varepsilon_{can} < -2$ MeV) corresponding to the "core", and the remaining weakly bound levels close to the threshold ($\varepsilon_{can} > -0.3$ MeV) and in the continuum corresponding to the "halo".

As discussed in Refs. [18, 19, 20], the shape of the halo originates from the intrinsic structure of the weakly bound or continuum orbitals. By examining the neutron density distribution, we learned that for the ground state of ^{42}Mg, the halo is mainly formed by level 4 and level 5. Note that the angular distribution of $|Y_{10}(\theta,\phi)|^2 \propto \cos^2\theta$ with a projection of the orbital angular momentum on the symmetry axis $\Lambda = 0$ is prolate and that of $|Y_{1\pm1}(\theta,\phi)|^2 \propto \sin^2\theta$ with $\Lambda = 1$ is oblate. For level 4 ($\Omega^\pi = 1/2^-$), Λ could be 0 or 1 since the third component of the total spin is $1/2$. However, it turns out that the $\Lambda = 1$ component dominates which results in an oblate shape. For level 5, since the
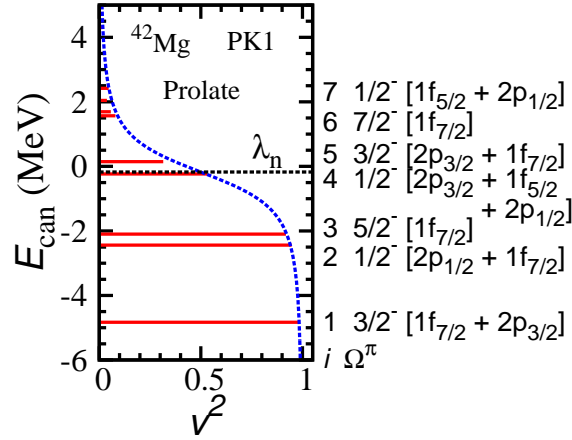
FIGURE 2. (Color online) Single neutron levels of ground state of ^{42}Mg in the canonical basis as a function of the occupation probability v^2. The blue dashed line corresponds to the BCS-formula with an average pairing gap. Taken from Ref. [20].

third component of the total spin is $3/2$, Λ can only be 1, which corresponds to an oblate shape too. Therefore in ^{42}Mg the shape of the halo is oblate and decouples from the prolate core.

The decoupling between the deformations of the core and the halo may manifest itself by some new experimental observables, e.g., the double-hump shape of longitudinal momentum distribution in single-particle removal reactions and new dipole modes, etc. In particular, a combination of the experimental method proposed in Ref. [28] and the theoretical approach developed in Ref. [29] would be useful in the study of longitudinal momentum distribution in single-particle removal reactions with deformed halo nuclei as projectiles. The shape decoupling effects may also has some influence on the sub-barrier capture process in heavy ion collisions [30].

For odd particle system, the formation and the size of a halo depend strongly on the interplay among the odd-even effects, continuum and pairing effects, deformation effects, etc. Some progress on this topic has been made recently [31].

SUMMARY

We present recent progresses of the development of a deformed relativistic Hartree-Bogoliubov theory in continuum (DefRHBC) and the study of neutron halo in deformed nuclei. In very neutron-rich deformed nuclei ^{42,44}Mg, pronounced deformed neutron halos were predicted. The halo is formed by several orbitals close to the threshold. These orbitals have large components of low ℓ-values and feel therefore only a small centrifugal barrier. Although their cores are prolately deformed, the deformation of the halos is slightly oblate. This implies a decoupling between the shapes of the core and the halo. The mechanism is investigated by studying the details of the neutron densities for core and halo, the single particle levels in the canonical basis, and the decomposition of the halo orbitals. It was concluded that the existence and the deformation of a possible neutron halo depends essentially on the quantum numbers of the main components of

the single particle orbits in the vicinity of the Fermi surface.

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